THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 2

1. Let $(X, \mathfrak{T}_X)$ and $(Y, \mathfrak{T}_Y)$ be two topological spaces. The product topology on $X \times Y$ is defined by

$$\mathfrak{T}_{X\times Y} = \text{the topology generated by the base } \{U \times V \mid U \in \mathfrak{T}_X, V \in \mathfrak{T}_Y\}$$

   Let $A \subset X$ and $B \subset Y$. Show that $\mathfrak{T}_X|_A \times \mathfrak{T}_Y|_B = \mathfrak{T}_{X\times Y}|_{A\times B}$.

2. Let $(X, \mathfrak{T}_X)$ be a topological space. Suppose that $A \subset Y \subset X$. Show that if $A \subset Y$ is closed in $(Y, \mathfrak{T}_X|_Y)$ and $Y \subset X$ is closed in $(X, \mathfrak{T}_X)$, then A is closed in $(X, \mathfrak{T}_X)$.

3. Let $f : (X, \mathfrak{T}_X) \to (Y, \mathfrak{T}_Y)$ be a surjective continuous function. Suppose D is a dense subset in X.
   (a) Show that $f(D)$ is also dense in Y.
   (b) * If E is a dense subset in Y, is $f^{-1}(E)$ necessarily dense in X?